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***Abstract*—In multi-rate IEEE 802.11 WLANs, the traditional user association based on the strongest received signal and the well known anomaly of the MAC protocol can lead to overloaded Access Points (APs), and poor or heterogeneous performance. Our goal is to propose an alternative game-theoretic approach for association. We model the joint resource allocation and user association as a matching game with complementarities and peer effects consisting of selfish players solely interested in their individual throughputs. Using recent game-theoretic results we first show that various resource sharing protocols actually fall in the scope of the set of stability-inducing resource allocation schemes. The game makes an extensive use of the Nash bargaining and some of its related properties that allow to control the incentives of the players. We show that the proposed mechanism can greatly improve the efficiency of 802.11 with heterogeneous nodes and reduce the negative impact of peer effects such as its MAC anomaly. The mechanism can be implemented as a virtual connectivity management layer to achieve efficient APs-user associations without modification of the MAC layer.**

***Index Terms*—Game theory, matching games, coalition games, stability, control, 802.11.**

I. Introduction

The IEEE 802.11 based wireless local area networks (WLANs) have attained a huge popularity in dense areas as public places, universities and city centers. In such environments, devices have the possibility to use many Access Points (APs) and usually a device selects an AP with the highest received Radio Signal Strength Indicator (best-RSSI association scheme). In this context, the performance of IEEE 802.11 may be penalized by the so called 802.11 *anomaly* and by an imbalance in AP loads (congestion). Moreover, some APs may be overloaded while others are underutilized because of the association rule.

In this paper, we consider a fully distributed IEEE 802.11 network, in which selfish mobile users and APs look for the associations maximizing their individual throughputs. The considered distributed association problem with selfish users naturally motivates us to adopt matching game theory. We thus analyze this scenario using matching games and develop a unified analysis of the mobile user association and resource allocation problem for the reduction of the anomaly and for load balancing in IEEE 802.11 WLANs. In a network characterized by a *state of nature* (user locations, channel conditions, physical data rates), composed of a set $\mathcal{W}$ of mobile users and a set $\mathcal{F}$ of APs, the user association problem consists in finding a mapping μ that associates every mobile user to an AP. We call the set formed by an AP and its associated mobile users a *cell*, or a *coalition* in the game framework. The set of coalitions induced by μ is called a matching or a *structure* (partition of the players in coalitions). Once mobile user association has been performed, a resource allocation scheme (also called a *sharing rule* in this paper) allocates radio resources of a cell to the associated mobile users.

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This matching game is characterized by *complementarities* in the sense that APs have preferences over groups of mobile users and *peer effects* in the sense that mobile users care who their peers are in a cell and thus emit preferences also over groups of mobiles users. Indeed, by definition of DCF implementation of the IEEE 802.11 protocol, a users' throughput does not only depend on its physical data rate but also on the coalition size and composition. The following questions are raised: are there stable associations? Do these associations always exist? Is there unicity? How to reach these equilibria in a decentralized way? Finally, how to provide the players the incentive to make the system converge to another association point with interesting properties in terms of load balancing?

In this game, some users may remain unassociated since every AP has the incentive to associate with a single user having the best data rate. To counter this so called *unemployment problem* and control the set of the stable matchings, we design a decentralized three steps mechanism. In the first step, the APs share the load. In the second step, the matching game is controlled to provide the incentives to enforce the load balancing. The control is based on the notion of Fear of Ruin (FoR) introduced in [6]. In the third step, players play the *controlled* matching game with individual payoffs obtained from a Nash Bargaining (NB) sharing rule. Under some assumptions, the NB sharing rule guarantees that the set of stable structures is non empty in all states of nature. A core stable matching is obtained by a decentralized algorithm. We propose here a modified version of the Deferred Acceptance Algorithm (DAA), called Backward Deferred Acceptance Algorithm (BDAA), for matching games with complementarities and peer effects. Similarly to the DAA, the complexity of the BDAA is polynomial in the number of proposals.

A. Related Work

IEEE 802.11 (WiFi) anomaly is a well documented issue in the literature, see e.g. [10], [11], [16]. Several approaches for a single cell modify the MAC so as to achieve a *time-based fairness* [10], [11] or *proportional fairness* [16]. Throughput based fairness, time based fairness and proportional fairness resource allocation schemes are sharing rules that can be obtained from a Nash bargaining as we will see later on.

In a multiple cell WLAN, mobile user-AP association plays a crucial role for improving the network performance and can be seen as a mean to mitigate the WiFi anomaly without modifying the MAC layer. The maximum RSSI association approach, though very simple, may cause an imbalanced traffic load among APs. Kumar et al. [14] investigate the problem of maximizing the sum of logarithms of the throughputs. Bejerano et al. [19] formulate a mobile user-AP association problem guaranteeing a max-min fair bandwidth allocation for mobile user. This problem is shown to be NP-hard and constant-factor approximation algorithms are proposed.

Arguing for ease of implementation, scalability and robustness, several papers have proposed decentralized heuristics to solve this issue, see e.g. [13], [22], [24]. Reference [13] proposes to enhance the basic RSSI scheme by an estimation of the Signal to Interference plus Noise Ratio (SINR) on both the uplink and the downlink. Bonald et al. in [24] show how performance strongly depends on the frequency assignment to APs and propose to use both data rate and MAC throughput in a combined metric to select the AP. Several papers have approached the problem using game theory based on individual MAC throughput. Due to the WiFi anomaly, this is not a classical *crowding game* in the sense that the mobile user achieved throughput is not necessary a monotonically decreasing function of the number of attached devices, as it can be the case in cellular networks [21], [25]. Compared to proposed decentralized approaches, we do not intend to optimize some network wide objective function, but rather to study the equilibria resulting from selfish behaviors. Compared to other game-theoretic approaches, we consider a fully distributed scenario, in which APs are also players able to accept or reject mobile users. This requires the study of the core stability, a notion stronger than the classical Nash Equilibrium. Moreover, there is a need in understanding the fundamental interactions between mobile user association and resource allocation in the presence of complementarities and peer effects.

In this paper, we tackle the mobile user-AP association problem using the framework of matching games with individual selfish players. This framework provides powerful tools for analyzing the stability of associations resulting from decentralized mechanisms. Matching games [7] is a field of game theory that have proved to be successful in explaining achievements and failures of matching and allocation mechanisms in decentralized markets. Gale and Shapley published one of the earliest and probably most successful paper on the subject [3] and solved the stable marriage and college admissions association problem with a polynomial algorithm called DAA.

Some very recent papers in the field of wireless networks have exploited the theoretical results and practical methods of matching games [28], [29], [30], [34], [35], although none has considered the WLAN association problem and its related WiFi anomaly. Authors of [28] address the problem of downlink association in wireless small-cell networks with device context awareness. The relationship between resource allocation and stability is not investigated and APs are not allowed to reject users. Hamidouche et al. in [29] tackle the problem of video caching in small-cell networks. They propose an algorithm that results in a many-to-many pairwise stable matching. Preferences emitted by servers exhibit complementarities between videos and vice versa. Nevertheless, the model doesn't take into account peer effects within each group. Reference [30] addresses the problem of uplink user association in heterogeneous wireless networks. Invoking a high complexity, complementarities are taken into account by a transfer mechanism that results in a Nash-stable matching, a concept weaker than pairwise stability or core stability. References [34], [35] study cognitive radio networks and do not exhibit complementarities or peer effects in the definition of players' preferences.

B. Contributions

Our contributions can be summarized as follows:

- We provide a matching game-theoretic unified approach of mobile user association and resource allocation in IEEE 802.11 WLANs in the presence of complementarities and peer effects. To the best of our knowledge, this is the first game-theoretic modeling of the IEEE 802.11 protocol covering such a number of resource allocation mechanisms proposed in the literature.
- We use existing theoretical results to show that if the scheduling and/or the MAC protocol result from a Nash bargaining then there exist stable mobile user associations, whatever the user data rates or locations. This result highlights the importance of the Nash bargaining in wireless networks as a stability inducer.
- In order to *control* the matching game, we design a three steps mechanism, which includes 1) a generic load balancing, 2) a control step, which modifies agents' preferences to provide them the incentive to enforce the result of the load balancing, 3) a matching game with resource allocation defined as a Nash bargaining and a stable matching algorithm. The control step tackles the so called unemployment problem, that would have left mobile users aside from the association otherwise. We show through numerical examples that our mechanism achieves good performance compared to the global optimum solution. To the best of our knowledge, such a

mechanism is absent from both the game theoretic and wireless networks based on matching games literature.

• We show that our BDAA can be efficiently used to find a stable many-to-one matching in a matching game with complementarities and peer-effects. The algorithm has a polynomial complexity in the number of proposals.

The mechanism has been originally proposed in an extended abstract [33]. Equal sharing has already been assessed in [32]. BDAA has been originally proposed in a short paper [31]. In this paper, we provide a complete description and show the mathematical results. We furthermore generalize the mechanism to a generic load balancing scheme and to the Nash bargaining-based sharing rules (resource allocation schemes). Finally, we show new numerical results.

The rest of the paper is organized as follows. In Section II, we define the system model. In Section III, we formulate the IEEE 802.11 WLANs resource allocation and decentralized association problem. In Section IV, we show that there exist stable coalition structures under certain conditions whatever the individual data rates. Section V presents our three steps mechanism. Section VI shows numerical results. Section VII concludes the paper and provides perspectives.

II. System model

We summarize in Table I the notations used in this paper. We use both game-theoretic definitions and their networking interpretation. Throughout the paper, they are used indifferently. Let define the set of players (nodes) $\mathcal{N}$ of cardinality N as the union of the disjoint sets of mobile users $\mathcal{W}$ of cardinality W and APs $\mathcal{F}$ of cardinality F. As in [14], we assume an interference-free model. It is assumed that the AP placement and channel allocation are such that the interference between cochannel APs can be ignored. In game-theoretic terms, this implies that there are no externalities. The mobile user association is a mapping μ that associates every mobile user to an AP and every AP to a subset of mobile users.

The IEEE 802.11 standard MAC protocol has been set up to enable any node in $\mathcal{N}$ to access a common medium in order to transmit its packets. The physical data rate between a transmitter and a receiver depends on their respective locations and on the channel conditions. For each mobile user $i \in \mathcal{W}$, let θ_{if} be the (physical) data rate with an AP f where $\theta_{if} \in \Theta = \{\theta^1, \ldots, \theta^m\}$, a finite set of finite rates resulting from the finite set of Modulation and Coding Schemes. If i is not within the coverage of f then $\theta_{if} = 0$. Given an association μ, let $\boldsymbol{\theta}_C = (\theta_{wf})_{(f,w)\in(C\cap\mathcal{F})\times(C\cap\mathcal{W})}$ denote the data rate vector of mobiles users in cell C served by AP f. Let $\mathbf{n_C}$ be the normalized composition vector of C, whose k-th component is the proportion of users in C with data rate $\theta_k \in \Theta$. Note that an AP is defined in the model as a player with the additional property of having maximum data rate on the downlink. Within each cell, a resource allocation scheme (e.g. induced by the CSMA/CA MAC protocol) may be formalized as a sharing rule over the resource to be shared in the cell. This resource may be the total cell throughput (as considered in the saturated regime) or the amount of radio resources in time or frequency in the general case. More precisely, a sharing rule is a set of functions $D = (D_{i,C})_{C\in\mathcal{C}, i\in C}$, where $D_{i,C}$ allocates a part of the resource of C to user $i \in C$. Equal sharing, proportional fairness, α-fairness are examples of sharing rules.

Assuming the IEEE 802.11 MAC protocol and the saturated regime, the overall cell resource of cell C is defined as the total throughput. It is a function of the composition vector $\mathbf{n_C}$ and of the cardinality $|C|$. We denote $r_{i,C}$ the throughput obtained by user i in cell C. From the game theoretic point of view, $r_{i,C}$ is understood as i's share of the worth of coalition C denoted $v(C)$. The function $v : \mathcal{C} \to \mathbb{R}$ is called the characteristic function of the coalition game and maps any coalition $C \in \mathcal{C}$ to its worth $v(C)$. Other MAC protocols and regime can however be modeled by this approach. For example time-based fairness proposed in the literature to solve the WiFi anomaly results from the sharing of the time resource. In this case, a user i gets a proportion $\alpha_{i,C}$ of the time resource, which induces a throughout of $\alpha_{i,C}\theta_{if}$, where f is the AP of C. It can be shown that time-based fairness results in a proportional fairness in throughputs.

III. Matching Games Formulation

A. Matching Games for Mobile User Association

In this paper, the mobile user association is modeled as a matching game (in the class of coalition games). The matching theory relies on the existence of individual order relations $\{\succeq_i\}_{i\in\mathcal{N}}$, called preferences, giving the player's ordinal ranking[1] of alternative choices. As an example, $w_1 \preceq_{f_1} [w_2, w_3] \preceq_{f_1} w_4$ indicates that the AP f_1 prefers to be associated to mobile user w_4 to any other mobile user, is indifferent between w_2 and w_3, and prefers to be associated to mobile user w_2 or w_3 rather than to be associated to w_1. Following the notations of Roth and Sotomayor in [7], let us denote $\mathbf{P}$ the set of preference lists $\mathbf{P} = (P_{w_1}, \ldots, P_{w_W}, P_{f_1}, \ldots, P_{f_F})$.

Definition 1 (Many-to-one bi-partite matching [7])**.** *A matching μ is a function from the set $\mathcal{W} \cup \mathcal{F}$ into the set of all subsets of $\mathcal{W} \cup \mathcal{F}$ such that:*
(i) $|\mu(w)| = 1$ for every mobile user $w \in \mathcal{W}$ and $\mu(w) = w$ if $\mu(w) \notin \mathcal{F}$;
(ii) $|\mu(f)| \leq q_f$ for every AP $f \in \mathcal{F}$ ($\mu(f) = \emptyset$ if f isn't matched to any mobile user in $\mathcal{W}$);
(iii) $\mu(w) = f$ if and only if w is in $\mu(f)$.

[1]In this paper, we use the Individually Rational Coalition Lists (IRCLs) to represent preferences. It can indeed easily be shown that other representations (additively separable preferences, B-preferences, W-preferences) are not adapted to our problem, see [26] for more details.

$\|set\|$	cardinality of the set set	$\mathcal{N}$	set of players (mobile users and APs)
$\mathcal{W}$	set of mobile users	$\mathcal{F}$	set of Access Points (APs)
$\mathcal{C}$	set of coalitions (cells)	$\mathcal{C}_f$	set of coalitions containing AP $f \in \mathcal{F}$
C	coalition (cell)	μ	matching (AP-mobile user association)
Θ	set of feasible data rates	θ_{wf}	data rate between w and f
$r_{i,C}$	throughput of node (user or AP) i in cell C	$\alpha_{i,C}$	proportion of resources (time, frequency) of i in cell C
D	sharing rule (resource allocation scheme)	$v(C)$	worth of coalition C
$s_{i,C}$	payoff of player i in coalition C	$u_i(.)$	utility function of player i
q_f	quota of AP f	χ_C	fear-of-ruin of coalition C
$\hat{q}_f$	target load of AP f	Ω	control function
$P(i)$	preferences list of player i over individuals	$P^{\#}(i)$	preferences list of player i over groups

TABLE I
NOTATIONS

Condition (i) of the above definition means that a mobile user can be associated to at most one AP and that it is by convention associated to itself if it is not associated to any AP. Condition (ii) states that an AP f cannot be associated to more than q_f mobile users. Condition (iii) means that if a mobile user w is associated to an AP f then the reverse is also true. In this definition, $q_f \in \mathbb{N}^*$ is called the *quota* of AP f and it gives the maximum number of mobile users the AP f can be associated to.

From now on, we focus on many-to-one matchings. In this setting, stability plays the role of equilibrium solution. In this paper, we particularly have an interest in the pairwise and core stabilities. For more details we refer the reader to the reference book [7]. We say that a matching μ is *blocked by a player* if this player prefers to be unmatched rather than being matched by μ. We say that it is *blocked by a pair* if there exists a pair of unmatched players that prefer to be matched together.

Definition 2 (Pairwise sability [7])**.** *A matching μ is* **pairwise stable** *if it is not blocked by any player or any pair of players. The set of pairwise stable matchings is denoted $S(\mathbf{P})$.*

Definition 3 (Domination [7])**.** *A matching μ' dominates another matching μ via a coalition C contained in $\mathcal{W} \cup \mathcal{F}$ if for all mobile users w and APs f in C, (i) if $f' = \mu'(w)$ then $f' \in C$, and if $w' \in \mu'(f)$ then $w' \in C$; and (ii) $\mu'(w) \succ_w \mu(w)$ and $\mu'(f) \succ_f \mu(f)$.*

Definition 4 (Weak Domination [7])**.** *A matching μ' weakly dominates another matching μ via a coalition C contained in $\mathcal{W} \cup \mathcal{F}$ if for all mobile users w and APs f in C, (i) if $f' = \mu'(w)$ then $f' \in C$, and if $w' \in \mu'(f)$ then $w' \in C$; and (ii) $\mu'(w) \succeq_w \mu(w)$ and $\mu'(f) \succeq_f \mu(f)$; and (iii) $\mu'(w) \succ_w \mu(w)$ for some w in C, or $\mu'(f) \succ_f \mu(f)$ for some f in C.*

Definition 5 (Cores of the game [7])**.** *The core $C(\mathbf{P})$ (resp. the core defined by weak domination $C_W(\mathbf{P})$) of the matching game is the set of matchings that are not dominated (resp. weakly dominated) by any other matching.*

In the general case, the core of the game $C(\mathbf{P})$ contains $C_W(\mathbf{P})$. When the game does not exhibit complementarities or peer effects, it is sufficient for its description that the preferences are emitted over individuals only. In the presence of complementarities or peer effects, players in the same coalition (i.e. the set of mobile users matched to the same AP) have an influence on each others. In such a case, the preferences need to be emitted over subsets of players and are denoted $P^{\#}$.

In the classical case of matchings with complementarities, the preference lists are of the form $\mathbf{P} = (P_{w_1}, \ldots, P_{w_W}, P^{\#}_{f_1}, \ldots, P^{\#}_{f_F})$, i.e., preferences over groups are emitted only by the APs (see the firms and workers problem in [7]). Moreover, it may happen that the preferences over groups may be *responsive* to the individual preferences in the sense that they are aligned with the individual preferences in the preferences over groups differing from at most one player. The preferences over groups may also satisfy the substitutability property. The substitutability of the preferences of a player rules out the possibility that this player considers others as complements.

Definition 6 (Responsive preferences [7])**.** *The preferences relation $P^{\#}(i)$ of player i over sets players is responsive to the preferences $P(i)$ over individual players if, whenever $\mu'(i) = \mu(i) \cup \{k\} \backslash \{l\}$ for l in $\mu(i)$ and k not in $\mu(i)$, then i prefers $\mu'(i)$ to $\mu(i)$ (under $P^{\#}(i)$) if and only if i prefers k to l (under $P(i)$).*

Before defining the substitutability property of preferences, we need to introduce the choice function Ch_i of a player i. Given any subset C of players, $Ch_i(S)$ is called the choice set of i in S. It gives the subset of players in C that player i most prefers.

Definition 7 (Substitutable preferences [7])**.** *A player i's ($i \in \mathcal{W} \cup \mathcal{F}$) preferences over sets of players has the property of substitutability if, for any set S that contains players k and l, if k is in $Ch_i(S)$ then k is in $Ch_i(S \backslash l)$.*

Considering preference lists of the form $\mathbf{P} = (P_{w_1}, \ldots, P_{w_W}, P^{\#}_{f_1}, \ldots, P^{\#}_{f_F})$ and assuming either responsive or substitutable strict preferences, we have the result that $C_W(\mathbf{P})$ equals $S(\mathbf{P})$ [7, p.174]. Any many-to-one matching problem with these properties has an equivalent one-to-one matching problem, which can be solved by considering preferences over individuals only. The set of pairwise stable matching is non-empty.

If the preferences are neither responsive nor substitutable, the equality $S(\mathbf{P}) = C_W(\mathbf{P})$ does not hold in general and the sets of pairwise, weak core and core stable matchings may be empty. An additional difficulty appears if the preferences over groups have to be considered on the mobile users side, i.e., if we have preference lists of the form $\mathbf{P} = (P^{\#}_{w_1}, \dots, P^{\#}_{w_W}, P^{\#}_{f_1}, \dots, P^{\#}_{f_F})$. Complementarities and peer effect may arise in both sides of the matching. The user association problem in IEEE 802.11 WLANs falls in this category because the performance of any mobile user in a coalition may depend on the other mobiles in the coalition. To break the indifference, we use the following rule: a mobile user prefers a coalition with AP with the lowest index and an AP prefers coalitions in lexicographic order of users indices.

To see that preferences may not be responsive, consider an example with only uplink communications, two APs f_1 and f_2 and three mobile users w_1, w_2, w_3 such that $\theta_{11} = 300$ Mbps, $\theta_{12} = \theta_{22} = 54$ Mbps, $\theta_{21} = \theta_{32} = 1$ Mbps. Assuming saturated regime and equal packet size, we can show that $P^{\#}(w_1) = f_1 \succ f_2 \succ \{w_3; f_1\} \succ \{w_2; f_2\} \succ \{w_2; f_1\} \succ \{w_3; f_2\}$, which is not responsive. In this example, we also see that substitutability is not even defined since every choice set is reduced to a singleton. After the game has been controlled according the proposed mechanism, preferences of w_1 can be modified as follows: $P^{\#}(w_1) = \{w_3; f_1\} \succ \{w_2; f_2\} \succ \{w_2; f_1\} \succ \{w_3; f_2\} \succ f_1 \succ f_2$. Considering $S = \{w_2, w_3; f_1, f_2\}$, we have $Ch_{w_1}(S) = \{w_3; f_1\}$, while $Ch_{w_1}(S \backslash w_3) = \{w_2; f_2\}$. Preferences are thus not substitutable.

This general many-to-one matching problem has algorithmically been assessed by Echenique and Yenmen in [17] who propose a fixed-point formulation and an algorithm to enumerate the set of stable matchings. It is known, that there is no guarantee that this set is non empty if the individual preferences over groups are not of a particular form. The problem of complementarities and peer effects in matchings has been analytically tackled by Pycia in [27]. Nevertheless, no result have been derived concerning the control of core stable structures and no matching algorithm with such reduced lists of preferences for the mobiles have been derived.

B. Sharing Rules and Matching Game Formulation

We now assume that a player i in a given coalition C obtains a *payoff* $s_{i,C}$, which is evaluated (or perceived) by it through a *utility function* $u_i : \mathbb{R} \to \mathbb{R}$. In this paper, we assume that functions u_i are positive, concave (thus log-concave), increasing and differentiable. In such a case, the individual preferences are induced by the player's utilities of these payoffs. Note that as utility functions are increasing, user's preferences are equivalently based on their payoff or their utility. We extend our model to the framework of finite coalition games in characteristic form $\Gamma = (\mathcal{N}; v)$, where v is a function mapping any coalition to its worth in $\mathbb{R}^+$. By definition of the characteristic function $v(\emptyset) = 0$. In this paper we do not assume a particular form of the characteristic function v (e.g. super-additivity[2]). An even particular case of coalition games in characteristic form concerns games with an exogenous sharing rule $\Gamma = (\mathcal{N}; v; E^N; D)$, where E^N is the set of all payoff vectors and D is a sharing rule.

Definition 8 (Sharing Rule)**.** *A sharing rule is a collection of functions $D_{i,C} : \mathbb{R}^+ \to \mathbb{R}^+$, one for each coalition C and each of its members $i \in C$, that maps the worth $v(C)$ of C into the share of output obtained by player i. We denote the sharing rule given by functions $D_{i,C}$ as $D = (D_{i,C})_{C \in \mathcal{C}, i \in C}$.*

From this definition, the payoff of user i in coalition C is given by $s_{i,C} = D_{i,C} \circ v(C)$ (where $\circ$ is the composition) and his utility of this payoff is given by $u_i(s_{i,C})$. We can now formulate the IEEE 802.11 joint user association and resource allocation problem as a matching game.

Definition 9 (Resource Allocation and User Association Game)**.** *Using the above notations, the resource allocation and users association game is defined as a N-player many-to-one matching game in characteristic form with sharing rule D and rates $\boldsymbol{\theta} = \{\theta_{wf}\}_{(w,f) \in \mathcal{W} \times \mathcal{F}}$: $\Gamma = (\mathcal{W} \cup \mathcal{F}, v, \mathbb{R}^{+N}, D, \boldsymbol{\theta})$. Each pair of players of the form $(w, f) \in \mathcal{W} \cup \mathcal{F}$ is endowed with a rate θ_{wf} from the rates space $\Theta = \{\theta^1, \dots, \theta^m\}$. For this game, we define the set of possible coalitions $\mathcal{C}$:*

$$\mathcal{C} = \{\{f\} \cup J,\ f \in \mathcal{F},\ J \subseteq \mathcal{W}, |J| \leq q_f\} \cup \{\{w\},\ w \in \mathcal{W}\}. \tag{1}$$

Note that for IEEE 802.11 MAC protocol and for the saturated regime, $s_{i,C} \triangleq r_{i,C}$. For other time sharing MAC approaches, $s_{i,C} \triangleq \alpha_{i,C}$.

IV. Existence of Core Stable Structures

A. Background on Nash Bargaining

The analytical theory of bargaining has mainly been developed on the concept introduced by J.F. Nash in [1] and [2] for the two person game and by Harsanyi in [4] for the N-person game. The bargaining is developed as a cooperative game where the set of acceptable (feasible) individual payoffs results from the set of mutual agreements among the players involved. In this paper, we use the Nash bargaining to model the resource allocation scheme (see Section V).

Let $\mathbf{t} = (t_1, \dots, t_N)$ be the fixed threat vector, an exogeneous parameter that may or not result from a threat game. The bargaining problem consists in looking for a payoff vector $(u_1, ..., u_N)$ satisfying five axioms: (i) Strong Efficiency, (ii) Individual Rationality, (iii) Scale Invariance, (iv) Independence of Irrelevant Alternatives and, (v) Symmetry.
We have the following result (see [15] and references therein):

[2] $\forall C, C,\ v(C \cup C') \geq v(C) + v(C')$ if $C \cap C' = \emptyset$

Theorem 1. *Let the utility functions u_i be concave, upper-bounded and defined on X, a convex and compact subset of $\mathbb{R}^n$. Let X_0 be the set of payoffs s.t. $X_0 = \{\mathbf{s} \in X | \forall i, u_i(s_i) \geq t_i\}$ and J be the set of users s.t., $J = \{j = \{1, \ldots, N\} | \exists \mathbf{s} \in X, s.t.\ u_j(s_j) > t_j\}$. Assume that $\{u_j\}_{j \in J}$ are injective. Then there exists a unique Nash bargaining problem as well as a unique Nash bargaining solution $\mathbf{s}$ that verifies $u_j(s_j) > t_j$, $j \in J$, and is the unique solution of the problem:*

$$\max_{\mathbf{s} \in X_0} \prod_{j \in J} (u_j(s_j) - t_j). \qquad (2)$$

An important result about the Nash bargaining solution is that it achieves a generalized proportional fairness which includes as a special case the well-known and commonly-used proportional fairness. The proportional fairness is achieved in the utility space with a null threat vector. Nevertheless, if the players' utility functions are linear in their payoffs and the threats vector is null, the proportional fairness is achieved in the payoff space. In other words, the payoff vector induced by the Nash Bargaining over the coalition throughput is proportional fair. This makes game-theory and in particular the bargaining problem of fundamental importance in networks. In Appendix A in [36], we show that the resource allocation induced by the MAC protocol of IEEE 802.11 can be modeled as the result of a Nash bargaining with power functions as utilities satisfying the conditions of Theorem 1.

B. On the Existence of Core Stable Structures

In this section, we show the existence of stable coalition structures (users-AP association) when preferences are obtained under some regularity conditions over the set of coalitions and some assumptions over the monotonicity of the sharing rules. There exists a stable structure of coalitions whatever the state of nature $\boldsymbol{\theta}$ if and only if the sharing rules may be formulated as arising from the maximization of the product of increasing, differentiable and strictly log-concave individual utility functions in all coalitions.

Proposition 2 ([27], Corollary 2)**.** *If the set of coalitions $\mathcal{C}$ is such that $q_f \in \{2, \ldots, W-1\}$ and $F \geq 2$, and if the sharing rule D is such that all functions $D_{i,C}$ are striclty increasing, continuous and $\lim_{y \to +\infty} D_{i,C}(y) = +\infty$, then there is a stable coalition structure for each preference profile induced by the sharing rule D iff there exist increasing, differentiable, and strictly log-concave functions $u_i : \mathbb{R}^+ \to \mathbb{R}^+, i \in \mathcal{N}$, such that $\frac{u_i(0)}{u_i'(0)} = 0$ and*

$$(D_{i,C} \circ v(C))_{i \in C} = \operatorname*{argmax}_{\mathbf{s}_C \in B_C} \prod_{i \in C} u_i(s_{i,C}), \qquad (3)$$

where $C \in \mathcal{C}$ and $B_C = \{\mathbf{s}_C = (s_{i,C})_{i \in C} | \sum_{i \in C} s_{i,C} \leq v(C)\}$.

According to Proposition 2, there are two conditions to be satisfied by the coalitions of our resource allocation and user association game: (i) consider scenarios with at least two APs (which is reasonable when talking about load balancing) and (ii) every AP is supposed to be able to serve at least two users and should not be able to serve the whole set of users. For more details, see Appendix B in [36].

Proposition 2 ensures that there exists a stable coalition structure as soon as resource allocation results from a Nash bargaining. The equal sharing resulting from CSMA/CA MAC protocol in saturated regime, single-flow per device and equal packet length is obtained by considering $s_{i,C} = r_{i,C}$ and the identity function for u_i. The players' throughputs in the general saturated regime with multiple flows and heterogenous packet length is obtained by taking $s_{i,C} = r_{i,C}$ and utility functions of the form $u_i(s_{i,C}) = s_{i,C}^{\alpha_i}$, where the bargaining power $\alpha_i \in [0, 1]$ is a function of the average packet length of user i (as shown in Appendix A in [36]). In these examples, utility functions u_i verify the conditions of Proposition 2.

In CSMA/CA under saturated regime, the cell throughput is increasing with the individual physical data rates and individual throughputs $r_{i,C}$ are sub-additive, i.e., decreasing with the addition of users. Assuming that the payoff is the individual throughput, i.e., $s_{i,C} = r_{i,C}$, then each player has the incentive to form the lowest cardinality coalition with highest composition vector. In this case, the unique stable structure is a one-to-one matching, in which APs are associated to their best mobile user. This will further be mentioned in the name of the *unemployment problem* since it leaves some mobiles users unassociated (unmatched). There is the need for a control of the players incentives for some equilibrium points with satisfying properties, in terms of unemployment in the present case. In other words, since the players have the incentive to match in a one-to-one form, one needs to control the underlying cooperative game so as to provide new incentives for a suitable many-to-one form as an equilibrium.

V. Mechanism for Controlled Matching Game

In order to tackle the *unemployment problem*, we propose in this section a mechanism to control the players incentives for coalitions (see Figure 1). This mechanism is made of three steps. We start by considering for every AP the set of acceptable mobile users, i.e., the mobile users with non zero data rate with this AP. In the first step (block **LB**), APs share the load defined in number of users. This results in target loads that should be enforced by the mechanism. The second step (blocks $\boldsymbol{\Omega}$ and $\boldsymbol{\Phi}$) is a controlled coalition game designed so as to provide the players the incentives to form coalitions with cardinalities given by the target loads and reducing heterogeneity (and thus reducing the anomaly in the IEEE 802.11). The third step (block μ) is a decentralized coalition formation (or matching) algorithm which results in a stable structure induced by the individual preferences influenced by the controlled coalitional game.

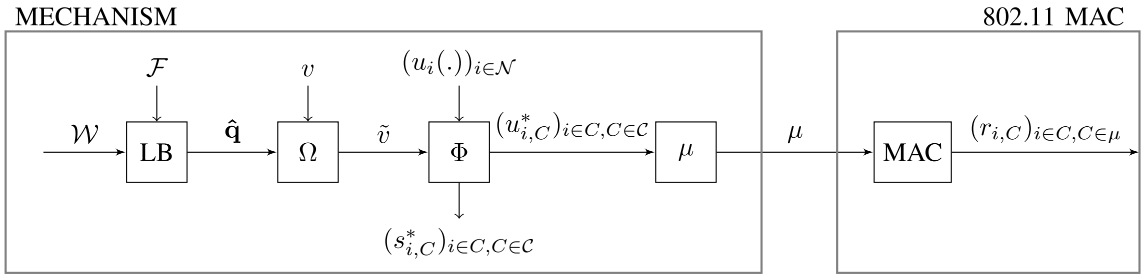


Fig. 1. Block diagram of the mechanism in the most general form. The APs share the load in the block **LB** which gives the APs' target loads $\hat{\mathbf{q}}$. The characteristic function v of the original coalition game is controlled in $\boldsymbol{\Omega}$ and gives the modified characteristic function $\tilde{v}$. The Nash bargaining $\boldsymbol{\Phi}$ is played in each coalition for the allocation of the worth of the coalition among its members. The players then emit their preferences over the coalitions on the basis of their shares and enter a stable matching mechanism in block μ. This block outputs an AP-user association μ. Finally, in the block $\mathbf{MAC}$ the nodes transmit their packets according to the unmodified IEEE 802.11 MAC protocol.

Our mechanism can be implemented as a virtual connectivity management layer on top of the IEEE 802.11 MAC protocol. Mobile users and APs form coalitions based on the ”virtual rates” provided by this virtual layer. Once associated, users access the channel using the unmodified 802.11 MAC protocol.

A. Load Balancing

The first step of the mechanism is a load balancing. This step outputs a target load vector of the form $\hat{\mathbf{q}} = (\hat{q}_1, \ldots, \hat{q}_F)$ that defines the size of the coalitions the players should be incentivized to form with each AP. In other words, $\hat{\mathbf{q}}$ gives the number of connections the players should be incentivized to create with each AP. In numerical implementation (Section VI), users covered by several APs are equally shared by these APs. Nevertheless, any load balancing scheme can be used in this mechanism. For example, there are decentralized schemes that converge to the Nash Bargaining solution that is known to achieve a generalized proportional fair allocation [8], [15]. At this stage, there is no incentive for agents to respect the target loads and if nothing else is done the unemployment problem remains. The target loads serve as input for the control step.

B. Controlling coalition game

The second step of the mechanism is the control of the coalition game. The control step of the mechanism tackles the problem of the control of the set of stable matchings. We observed that when a coalition game is defined by a characteristic function and a sharing rule inducing sub-additive strictly positive individual payoffs (except for coalitions of size one or those containing players with zero data rates), the stable structures to be formed are made of coalitions of size two. This step of the mechanism develops an analytical framework and methodology for the control of the equilibria by the way of a control over the players' incitations for individual strategies.

Definition 10 (Controller)**.** *The controller is any entity (player or other) having the legitimacy and ability to change the definition of the game (players, payoffs, worths, information, coalitions).*

The *controller* may not be taking part in the game (e.g. the network operator in a wireless network, the government for a firms and workers association problem) or any player of the game with some kind of additional decisional abilities. In other words, it may be any entity having the ability to create or modify the individual incitations of the players for some strategy and thus the ability to change the definition of the game. These changes in the definition of the game in view of manipulating the players' equilibria strategies are called control transformation,

Definition 11 (Control transformation)**.** *A* ***control transformation*** Ω *is a mapping from the set of coalition games in characteristic form in itself.*

In the purpose of this paper it is sufficient to restrict the definition of the control transformations to the domain of coalition games in characteristic form. In fact, we further assume that the controller cannot arbitrarily move from one game to another without constraints. We assume that he or she can influence the equilibria by partial changes in the definition of the game (characteristic function, individual payoffs, ...) but can neither change the fundamental rules of the game (e.g. the rules of matching games) nor some essential elements such as the players taking part in the game or their strategy spaces. If Γ is a coalition game in characteristic form, then $\Omega(\Gamma)$ is a coalition game in characteristic form modified by the controller according to its (constrained) abilities. The limits of the abilities of such a controller are to be chosen by the game theorist or the designer of the mechanism so as to satisfy the fundamental hypothesis and description of the system he is looking at. As an example of work on the design of an incitations operator, Auman and Kurz [6] assess the problem of designing the joint taxation and redistribution scheme in the framework of a political majority-minority game. The majority is the controller and the incitations are induced by a multiplicative tax over the worths of the coalitions.

In Appendix C in [36], we give two simple examples of the mechanism we propose to control the player's individual incentives.

We now search for operators modifying the characteristic function v so as to provide players the incentives to form stable structures with coalitions of sizes $\hat{\mathbf{q}}$.

An important lever for controlling our matching game and designing operator $\mathbf{\Omega}$ is the fear-of-ruin (FoR). Formally, the FoR of user i in coalition C is defined as:

$$\chi_i(s_{i,C}) \triangleq \frac{u_i(s_{i,C})}{u_i'(s_{i,C})}. \tag{4}$$

The FoR of coalition C is obtained as the inverse of the Lagrange multiplier associated to the constraint $\sum_{i\in C} s_{i,C} \leq v(C)$ at the optimum of the Nash bargaining optimization problem (3). Two interesting characteristics of the FoR are that (i) in a coalitional game with Nash bargaining as sharing rule, the FoR is constant over the players in a coalition, i.e., $\chi_i(s_{i,C}) = \chi_C \ \forall i \in C$ at the bargaining solution point $s_{i,C}$ and (ii) with concave increasing utility functions, the individual payoffs increase in the common FoR [27]. Thus, the players have the incentives to form coalitions maximizing their FoR. In terms of control opportunities, this introduces the FoR as a lever to control the set of individual payoff-based incentives for coalitions. As an example, assume two coalitions C and C' and their FoRs: $\chi_C < \chi_{C'}$. Players in $C \cap C'$ prefer C' to C. Changing the values of the FoRs to obtain $\chi_C > \chi_{C'}$ changes the individual incentives of these players so that they now prefer C to C'.

Proposition 3. *Assume a coalition game $\Gamma = (\mathcal{F} \cup \mathcal{W}, v, \{u_i\}_{i\in N})$ in characteristic form with the Nash bargaining sharing rule over $v(C)$ for every coalition C in $\mathcal{C}$. Furthermore assume strictly increasing and concave utility functions*[3] *$u_i : \mathbb{R}^{\cdot+} \to \mathbb{R}^+, i \in \mathcal{N}$. The set of transformations Ω from the set of characteristic functions in itself that provide the players the incentive for some subset $\mathcal{C}'$ of coalitions in $\mathcal{C}$ must satisfy:*

$$F_{C'} \circ \Omega(v)(C') < F_C \circ \Omega(v)(C) \quad \forall C' \in \mathcal{C}', \forall C \in \mathcal{C}\backslash\mathcal{C}' \tag{5}$$

s.t. $C' \cap C \neq \emptyset$ and where $F_C = \left(\sum_{i\in C}\left(\frac{u_i'}{u_i}\right)^{-1}\right)^{-1}$, where g^{-1} denotes the inverse function of g.

Proof. See Appendix D in [36] . □

In order to derive our last result, we need to define the concept of single-peaked preferences. Let $X = \{x_1, \ldots, x_n\}$ denote a finite set of alternatives, with $n \geq 3$.

Definition 12 (Peak of preferences, [23])**.** *A preference relation $\succ$ on X is a linear order on X. The peak of a preference relation $\succ$ is the alternative $x^* = peak(\succ)$ such that $x^* \succ x$ for all $x \in X\backslash\{x^*\}$.*

Definition 13 (Single-Peaked preferences, [23])**.** *An axis O (noted by $>$) is a linear order on X. Given two alternatives $x_i, x_j \in X$, a preference relation $\succ$ on X whose peak is x^*, and an axis O, we say that x_i and x_j are on the same side of the peak of $\succ$ iff one of the following two condition is satisfied: (i) $x_i > x^*$ and $x_j > x^*$; (ii) $x^* > x_i$ and $x^* > x_j$.*
A preference relation $\succ$ is single-peaked with respect to an axis O if and only if for all x_i, $x_j \in X$ such that x_i and x_j are on the same side of the peak x^* of $\succ$, one has $x_i \succ x_j$ if and only if x_i is closer to the peak than x_j, that is, if $x^* > x_i > x_j$ or $x_j > x_i > x^*$.

We use the discrete version of this definition over $\mathbb{N}^+$. We immediately obtain the following corollary, which iteratively uses the condition of Proposition 3 to provide players the incentive for coalitions of size $\hat{q}_f$ with AP f.

Corollary 1. *Assume a coalition game $\Gamma = (\mathcal{F} \cup \mathcal{W}, v, \{u_i\}_{i\in N})$ in characteristic form with the Nash bargaining sharing rule over the $v(C)$ in every coalition $C \in \mathcal{C}$. Furthermore assume strictly increasing and concave utility functions $u_i : \mathbb{R}^+ \to \mathbb{R}^+, i \in \mathcal{N}$. The set of transformations Ω from the set of characteristic functions in itself that induce single-peaked preferences (peak at $\hat{q}_f$) in cardinalities over the set of coalitions with an AP $f \in \mathcal{F}$ must satisfy:*

$$\max_{\substack{C\in\mathcal{C}_f \\ s.t. |C|=q}} F_C \circ \Omega(v)(C) < \min_{\substack{C\in\mathcal{C}_f \\ s.t. |C|=q+1}} F_C \circ \Omega(v)(C), \quad \forall q \geq \hat{q}_f \tag{6}$$

and

$$\max_{\substack{C\in\mathcal{C}_f \\ s.t. |C|=q}} F_C \circ \Omega(v)(C) < \min_{\substack{C\in\mathcal{C}_f \\ s.t. |C|=q-1}} F_C \circ \Omega(v)(C), \quad \forall q \leq \hat{q}_f \tag{7}$$

where $F_C = \left(\sum_{i\in C}\left(\frac{u_i'}{u_i}\right)^{-1}\right)^{-1}$.

Proof. See Appendix D in [36] . □

Note that the control step can be centralized or decentralized. In a decentralized implementation, every AP applies the control independently according to its target load by modifying the worth of the coalitions with this AP.

C. Access Point Association

The third step of the mechanism is the joint resource allocation and users association (matching) game where the players (APs and mobile users) share the resource in the coalitions according to a Nash bargaining and then match with each other. The coalition game played has been described in Section III and Section IV. This step corresponds to the blocks Φ and μ of the block diagram in Figure 1.

1) Stable Matching Mechanism: We now show that a modified version of the Gale and Shapley's deferred acceptance algorithm in its college-admission form with

[3] Such utility functions are bijective and thus injective. Theorem 1 applies.

Algorithm 1: Backward Deferred Acceptance

Data: For each AP: The set of acceptable (covered) users and AP-user data rates.
For each user: The set of acceptable (covering) APs.
Result: A core stable structure $\mathcal{S}$

begin
- *Step 1: Initialization*;
  - **Step 1.a:** All APs and users are marked *unengaged*. $L(f) = L^*(f) = \emptyset, \forall f$;
  - **Step 1.b:** Every AP f computes possible coalitions with its acceptable users, the respective users payoffs and emits its preference list $P^{\#}(f)$;
  - **Step 1.c:** Every AP f transmits to its acceptable users the highest payoff they can achieve in coalitions involving f;
  - **Step 1.d:** Every user w emits its reduced list of preference $P'(w)$;
- *Step 2 (BDAA)*;
  - **Step 2.a, Mobiles proposals:** According to $P'(w)$, every unengaged user w proposes to its most preferred acceptable AP for which it has not yet proposed. If this AP was engaged in a coalition, all players of this coalition are marked *unengaged* ;
  - **Step 2.b, Lists update:** Every AP f updates its list with the set of its proposers: $L(f) \longleftarrow L(f) \cup \{\text{proposers}\}$ and $L^*(f) \longleftarrow L(f)$;
  - **Step 2.c, Counter-proposals:** Every AP f computes the set of coalitions with users in the dynamic list $L^*(f)$ and counter-proposes to the users of their most preferred coalition according to $P^{\#}(f)$;
  - **Step 2.d, Acceptance/Rejections:** Based on these counter-proposals and the best achievable payoffs offered by APs in Step 1.c to which they have not yet proposed, users accept or reject the counter-proposals;
    - **Step 2.e:** If all users of the most preferred coalition accept the counter-proposal of an AP f, all these users and f defect from their previous coalitions;
    - all players of these coalitions are marked *unengaged*;
    - users that have accepted the counter-proposal and f are marked *engaged in this new coalition*;
    - **Step 2.f:** Every unengaged AP f updates its dynamic list by removing users both having rejected the counter-proposal and being engaged to another AP:
    - $L^*(f) \longleftarrow L^*(f) \backslash \{\text{engaged rejecters}\}$;
  - **Step 2.g:** Go to Step 2.c while the dynamic list L^* of at least one AP has been strictly decreased (in the sense of inclusion) in Step 2.f;
  - **Step 2.h:** Go to Step 2.a while there are unengaged users that can propose;
  - **Step 2.i:** All players engaged in some coalition are matched.

APs preferences over groups of users and users preferences over individual APs is a stable matching mechanism for the many-to-one matching games with complementarities, peer effects considered in this paper (see Algorithm 1: Backward Deferred Acceptance).

BDAA is similar to the DAA in many aspects. It involves two sets of players that have to be matched. Every player from one side has a set of unacceptable players from the other side. In our case, an AP and a mobile user are acceptable to each others if the user is under the AP coverage. As in DAA, the algorithm proceeds by proposals and corresponding acceptances or rejections. The main difference resides in the notion of counter-proposals, introduced to tackle the problem of complementarities.

The block diagram representation of the algorithm is shown in Figure (2). In block $\mathbf{P}^{\#}$ the APs emit their preferences over the coalitions. In block $\mathbf{P}^{'}$ the mobiles emit their preferences over the APs. In block *Proposals* the mobiles propose to the APs. In block *counter-proposals* the APs counter-propose. The counter-proposing round continues up to convergence. The next proposing round starts.

We enter the details of the algorithm. Having the information of the data rates with users under their coverage, APs are able to compute all the possible coalitions they can form and the corresponding allocation vectors (throughputs). They can thus build their preference lists (Steps 1b). Then, every AP f transmits to each of its acceptable users the maximum achievable throughput (based on MAC layer and virtual mechanism) it can achieve in the coalitions it can form with f (Step 1.c). Every user w can thus build its reduced list of preferences over individual APs: w prefers f_i to f_j if the maximum achievable throughput with f_i is strictly greater than its maximum achievable throughput with f_j (Step 1.d). BDAA then proceeds by rounds during which users make proposals, AP make counter-proposals and users accept or reject (from Step 2.a to Step 2.h). Every AP that receives a new proposal shall reconsider the set of its opportunities and is thus marked unengaged (Step 2.a). $L(f)$ is the list of all users that have proposed at least once to AP f. $L^*(f)$ is a dynamic list that is reinitialized to $L(f)$ before every AP counter-proposal (Step 2.b). In each round of the algorithm, every unengaged user proposes to its most preferred AP for which it has not yet proposed (Step 2.a). Every AP receiving proposals adds the proposing players to its cumulated list of proposers and reinitializes its dynamic list (Step 2.b). Using $P^{\#}(f)$ it then searches for its most preferred coalition involving only users from the dynamic list and emits a counter-proposal to these users. This counter-proposal contains the throughput every user can achieve in this coalition (Step 2.c). Each user compares the counter-proposals it just received with the best achievable payoffs obtained with the APs it has not proposed to yet (Step 2.d). If one of these best achievable payoffs is strictly greater than the best counter-proposal, the users rejects the counter-proposals and continues proposing (Step 2.d, Step 2.h). Otherwise, the user accepts its most preferred counter-proposal (Step 2.d). Given a counter-proposal, if all users accept it, then they are engaged to the AP. All coalitions in which these users and the AP were engaged are broken and their players are marked unengaged (Step 2.e). If at least one user does not agree, then the AP is unengaged (Step 2.e), it updates its dynamic list by removing the mobiles having rejected its counter-proposal and being engaged to another AP (Step 2.f). The counter-proposals continues up to the point when no AP can emit any new counter-proposal (Step 2.g). The current round ends and the algorithm enters a new round (Step 2.h). The algorithm stops when no more users are rejected (Step 2.h). A stable matching is obtained (Step 2.i).

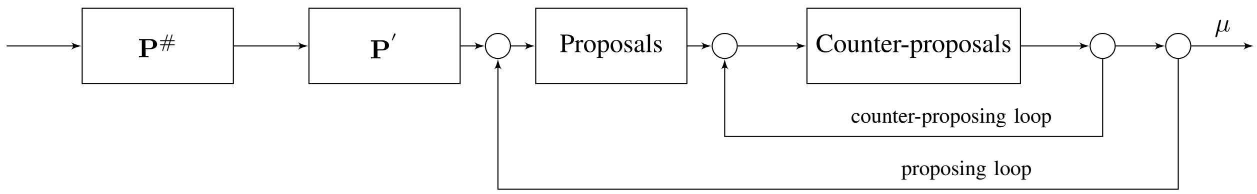


Fig. 2. Block diagram of the BDAA.

Remark: In Step 1.d, users' reduced preferences are optimistic preferences providing the users the incentive to propose to new APs as long as their corresponding best case is better than the received proposals.

Remark: We see from the above description that there are mainly two mechanisms that allow to take into account complementarities and peer effects. First, information is transferred from APs to users via throughput values and every throughput computed by an AP takes into account the composition of the cell and so the complementarities and peer effects between the nodes of this coalition. Second, an AP is always able to reconsider all possible coalitions with users having proposed to it so far, even those that have been rejected before. This is the main difference with DAA, where a rejected user is never reconsidered because of the absence of group effects.

Proposition 4. *Given a many-to-one matching game, BDAA converges, i.e., outputs a matching in a finite number of steps.*

Proof. See Appendix D in [36]. □

Proposition 5. *Suppose the family of coalitions $\mathcal{C}$ as defined in* (1)*, and a sharing rule as defined in proposition 2. Furthermore assume a tie-breaking rule such that there is no indifference (strict preferences). BDAA converges to the unique core stable matching.*

Proof. See Appendix D in [36]. □

Proposition 6. *The complexity of BDAA is $O(n^5)$ in the number of proposals of the players, where $n = \max(F, W)$.*

Proof. First we give an upper bound on the number of proposals emitted by the mobile users, then we give an upper bound on the number of proposals emitted by the APs. In at most F proposals, every mobile user has proposed to all the APs. Thus, in at most $F \times W$ proposals, the mobile users have proposed to all the APs. At a given round of proposals, the counter-proposals ends at Step 2.g when no dynamic list has been strictly decreased. These lists contains at most W users. In the worst case, a single dynamic list is decreased by one user, and thus after $W \times F$ rounds of counter-proposals, all dynamic lists are empty. The round of counter-proposals stops. At each round of counter-proposals, F APs counter-proposes. Thus, in at most $F \times W \times F$ the APs have emitted all their counter-proposals. We obtain that the total number of proposals (both mobile users proposals and APs counter-proposals) cannot exceed $F^3 \times W^2$. The complexity of BDAA is $O(n^5)$ where $n = \max(F, W)$. □

In Appendix E in [36], we provide an interpretation of BDAA in the economic framework. In Appendix F [36], we give an example of application of the BDAA.

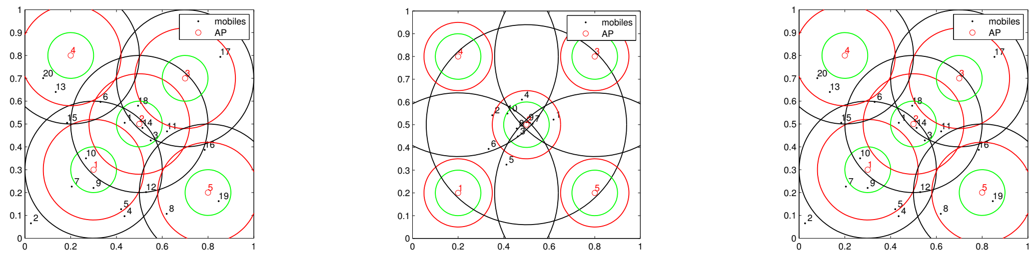
Remark: Note APs perform at initialization and at every counter-proposal search and sort operations on the set of coalitions. As the number of possible coalitions is $O(2^W)$ in the worst case, exponential complexity in number of elementary operations arises in the general case (a well known problem in coalition games). However, with dynamic lists, an AP considers only coalitions with users that have proposed to it so far. This reduces the dimension of the search space in practical implementations. Also, in the special case of WiFi, best coalitions are obtained with maximum data rates users so that there is no need to sort all coalitions to find the best. In the special case of WiFi with the proposed control in Corollary 1, coalitions of size closer to the target load are always preferred and within a class of equal size coalitions, those with best data rate users are preferred. Again, the sorting of all coalitions is not required. In the two WiFi cases, AP operations on coalitions are thus polynomial.

Remark: User association is decentralized in the sense that both APs and mobile users take decisions based on their preferences. As in DAA, the association can be performed by a central entity but it is not necessary. The resource allocation phase is the results of the WiFi MAC protocol, which is decentralized. In terms of information, every AP only needs to know the coalitions to which it can participate at initialization. In WiFi, only users under coverage are involved so that the needed information remains local.

VI. Numerical Results

A. Simulations Parameters and Scenarios

The numerical computations are performed under the assumption of equal packet sizes and saturated queues. Under this assumption the sharing rule is equal sharing. Analytical expressions of the throughputs (individual and total throughputs) are taken from [18] with the parameters of Table II. We further assume that a node compliant with a IEEE 802.11 standard (in chronological order: b, g, n) is compliant with earliest ones. By convention, if all nodes of a cell have the same data rate, we use the MAC parameters of the standard whose maximum physical



(a) Scenario 1. (b) Scenario 2. (c) A stable matching.

Fig. 3. (a) Scenario 1: A spatial distribution of APs (smallest red circles) $\mathcal{F} = \{f_1, \ldots, f_5\}$ and devices (black points) $\mathcal{W} = \{w_1, \ldots, w_{20}\}$. (b) Scenario 2: A spatial distribution of APs (smallest red circles) $\mathcal{F} = \{f_1, \ldots, f_5\}$ and devices (black points) $\mathcal{W} = \{w_1, \ldots, w_{10}\}$. Circles show the coverage areas corresponding to different data rates. (c) A stable matching in the uncontrolled case for Scenario 1.

	802.11n	802.11g	802.11b	
Parameter	value			unit
Θ	{300, 54, 11}	{54, 11}	{11}	Mbits/s
slot duration	9	9	20	μs
T_0	3	5	50	slots
T_C	2	10	20	slots
L	8192	8192	8192	bits
K	2	2	2	
b_0	16	16	16	
p	2	2	2	

TABLE II
SIMULATION PARAMETERS.

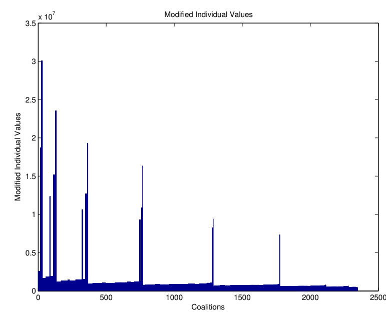
data rate is the common data rate. Otherwise, we use the MAC parameters of the standard whose maximum physical data rate is the lowest data rate in the cell.

Assume the spatial distributions of nodes of Figures 3(a) and (b). The first scenario (a) shows the case of 5 APs with a uniform spatial distribution of 20 mobile users. The second scenario (b) has non-uniform distribution of 10 mobile users in the plane. The green (inner), red (intermediate) and black (outer) circles show the spatial region where the mobiles achieve a data rate of 300 Mbits/s, 54 Mbits/s and 11 Mbits/s respectively. Scenario 2 exhibits a high overlap between AP coverages.

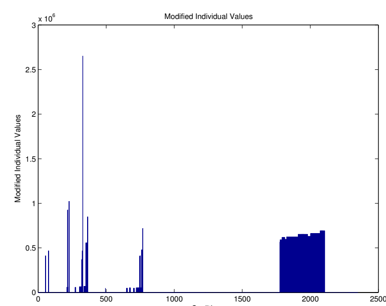
B. Numerical Work

1) No mechanism: We show in Figure 3(c) a stable matching. No associated player has an incentive to deviate and form a coalition of size superior to two. The figure shows the natural incentives of the system in forming low cardinalities coalitions with good compositions. This can also be observed on Figure 4 which shows the individual throughputs obtained in the coalitions. The coalitions are sorted by cardinalities from low to high. In plot (a) no mechanism is used. In plot (b) a gaussian tax rate is applied. See Section VI-B2.

Figure 3(c) and Figure 4(a) show the natural incentives of the system in forming low cardinalities coalitions with good compositions. As a result, a one-to-one matching is obtained. Using our mechanism, this structure of throughputs will be changed (as in Figure 4(b)) to move the incentives according to $\hat{\mathbf{q}}$ and thus provide the



(a) Individual throuputs vs. coalition index.



(b) Modified Individual throuputs vs. coalition index.

Fig. 4. (a) Scenario 1. Structure of the payoffs in the uncontrolled matching game. (b) Scenario 1. Structure of the payoffs in the controlled matching game with a multiplicative tax rate of variance $\sigma_f = 0.3, \forall f \in \mathcal{F}$.

players the incentives to associate according to a many-to-one matching rather than a one-to-one.

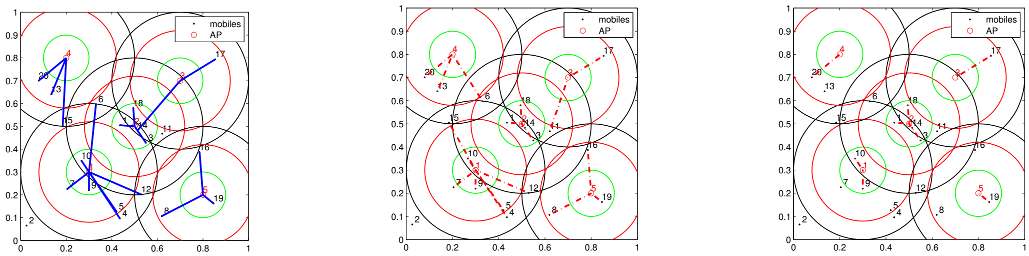
2) Gaussian Tax Rate in Cardinalities: As an example of family of cost functions, we can use multiplicative symmetric unimodal cost functions. The multiplicative cost functions are commonly called tax rates and are defined such that for any AP $f \in \mathcal{F}$ and any coalition C containing f, we must have:

$$\tilde{v}(C) = \mathbf{\Omega}(v(C)) \triangleq c_f(|C|)v(C) \tag{8}$$

We particularly consider Gaussian tax rates such that:

$$\tilde{v}(C) = e^{-\frac{(|C| - \hat{q}_f)^2}{2\sigma_f^2}} v(C) \tag{9}$$

where σ_f is the variance of the function c_f. The Gaussian cost function is convenient in the sense that it does not penalize the mean-sized coalitions and it provides a



(a) Stable matching resulting from Gaussian costs and BDAA.

(b) A global optimum association with Gaussian costs.

(c) A global optimum association without costs.

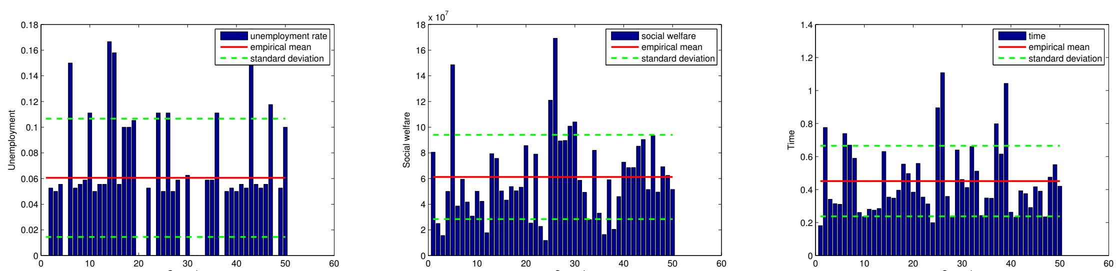
Fig. 5. Controlled matching game in scenario 1. Comparison of the association obtained from (a) BDAA, (b) a global optimum for Gaussian costs with variance $\sigma = 0.2$, (c) a global optimum without costs.

great amount of flexibility by the way of its variance. Decreasing or increasing the variance σ_f indeed allows for a strict or relaxed control of the incentives for the target loads. Focusing on the first scenario (Figure 3 (a)), we consider the three matchings shown in Figure 5. The first one (a) is the stable matching resulting from the mechanism (including BDAA and Gaussian costs); The second matching (b) maximizes the sum of modified throughputs (i.e. including Gaussian costs); The third matching (c) maximizes the sum of unmodified throughputs (i.e. without costs).

We first observe that the proposed mechanism induces a stable matching with a drastic reduction of the unemployment problem w.r.t. the result of Figure 3(c). The natural incentives of the system resulting in a one-to-one matching have been countered and a many-to-one matching is obtained. The unemployment has been reduced from 73% to 5% in this particular scenario. The second point to be raised is that the proposed mechanism allows to obtain (with a polynomial complexity) a stable matching with a high modified total throughput, close to the optimal modified total throughput that is however not stable. For this scenario, we achieve through our mechanism 99% of the total modified maximum throughput (see Figure 5(b)). This means that the cost for stability is very small in this particular scenario. Furthermore, the total throughput performance of the system at the MAC layer (i.e. unmodified throughputs obtained in block MAC of the block diagram representation of the mechanism, see Figure 1) is 97% the total unmodified maximum throughput (see Figure 5(b)) and 47% of the total maximum throughput of the uncontrolled system (see Figure 5(c)). This quantifies the cost for control, stability and low unemployment in this scenario. The third point is that the target loads have been enforced by the mechanism (via the cost function) since the target load vector is $\hat{\mathbf{q}} = (8.0, 4.5, 3.33, 3.83, 4.33)$ (obtained by Nash bargaining[4] over the share $[0, 1]$ of the players at the intersection of the coverages of the APs) and the formed coalitions are of sizes 8, 4, 3, 4 and 4.

We go into more details on the difference between the target load vector and the integer-sized coalitions in the stable matching. Focusing on AP3 with the target load $\hat{q}_3 = 3.33$, one may observe that in case of a Gaussian cost function with unit variance, the condition for an integer target load 3 is only satisfied for sizes of coalitions superior or equal to 4. This meaning that the use of a gaussian cost function centered on 3.33 and unit variance even though increasing the penalty with the distance in sizes to $\hat{q}_f$ cannot guarantee the systematic incentive to form coalitions of size 3 with AP3. There exists some coalitions of size 2 giving the players more individual throughputs than the worst coalition of size 3. In such case, the players will have the incentive to form

[4]Achieves a proportional fair allocation in the utility space of the APs. Induces the number of players to be connected to each AP.

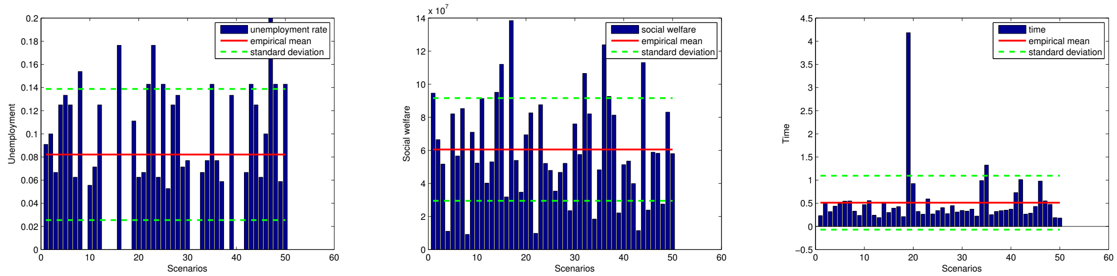


(a) Unemployment rates

(b) Modified social welfares

(c) Computation time of BDAA

Fig. 6. (a)Unemployment rates, (b)social welfares (the social welfare of a matching is measured as the total throughput of the system at equilibrium) and (c)computation times of BDAA over a sample of 50 scenarios obtained by spatial random uniform distribution of the mobile devices. APs are spatially distributed as in Scenario 1. For each plot, the red line gives the empirical mean $\hat{m}$ of the sample and the green dotted lines the interval $[\hat{m} - \sigma, \hat{m} + \sigma]$ where $\hat{m}$ is the empirical mean of sample and σ is the standard deviation.



(a) Unemployment rates

(b) Modified social welfares

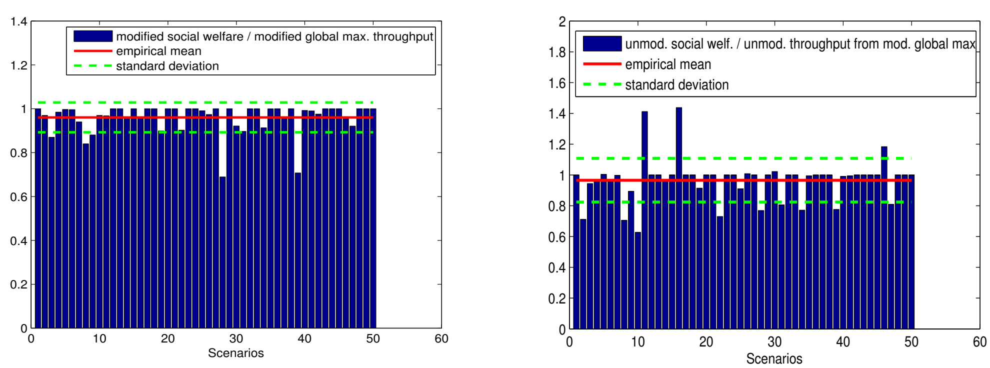
(c) Computation times of BDAA

Fig. 7. (a)Unemployment rates, (b)social welfares and (c)computation times of BDAA over a sample of 50 random networks obtained by spatial random uniform distribution of the mobile devices and APs. For each plot, the red line gives the empirical mean $\hat{m}$ of the sample and the green dotted lines the interval $[\hat{m} - \sigma, \hat{m} + \sigma]$ where $\hat{m}$ is the empirical mean of sample and σ is the standard deviation.

the coalition with the highest individual value among those of cardinalities 2 and 3. In Figure 6, we show the performance of the mechanism over a set of 50 scenarios generated by spatial random uniform distribution of the mobile devices. The APs are spatially distributed as in Scenario 1 (see Figure 7 for a random distribution of both the mobile devices and APs). The red line shows the empirical mean of the sample and the green dotted lines show the interval $[\hat{m} - \sigma, \hat{m} + \sigma]$ where $\hat{m}$ is the empirical mean of sample and σ is the standard deviation. The empirical mean of the unemployment rate is 6%, the mean modified social welfare is 61Mbits/s and the mean computation time of BDAA is 0.45s. Observe that in 22% of the realizations the unemployment is null and that in 70% of the realizations it is below the mean. In terms of computation times of BDAA, the mean performance is reasonably low (0.45s to match 20 mobiles to 5 APs) and the algorithm performs even better in 62% of the scenarios.

In Figure 7, we show the performance of the mechanism over a set of 50 scenarios generated by spatial random uniform distribution of the mobile devices and APs. The red lines show the empirical mean of the sample and the green dotted lines show the interval $[\hat{m} - \sigma, \hat{m} + \sigma]$ where $\hat{m}$ is the empirical mean of sample and σ is the standard deviation. The empirical mean of the unemployment rate is 8%, the mean modified social welfare is 60Mbits/s and the mean computation time of BDAA is 0.51s. Observe that in 22% of the realizations the unemployment is null and that in 56% of the realizations it is below the mean. In terms of computation times of BDAA, the mean is higher than in the previous case but the algorithm performs better than the mean in 68% of the scenarios.

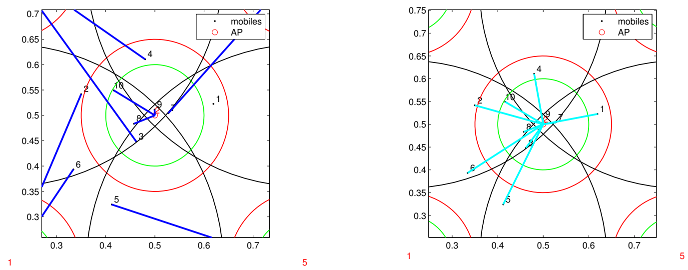
In Figure 8, plot (a), we show the ratios of the modified (mechanism level) social welfare (by definition, the total throughput resulting from BDAA) to the maximum total modified throughput. The mean performance of BDAA achieves 96% of this maximum. Furthermore, observe that the global maximum is achieved by BDAA in 46% of the random networks. The ratio is below $\hat{m} - \sigma$ in only 10% of the cases. In Figure 8, plot (b), we show the ratios of the unmodified (MAC level) social welfare to the unmodified total throughput induced at the matching maximizing the total modified throughput. The mean performance of BDAA achieves 97% of this unmodified total throughput. Finally, observe that in some cases, the ratio is even higher than one. This



(a) Ratios of modified performances

(b) Ratios of unmodified performances

Fig. 8. (a)Ratios of the modified social welfares to the maximum modified (mechanism level) total throughput, (b)Ratios of the unmodified (MAC level) social welfares to the unmodified total throughputs corresponding to the matching with maximum modified total throughput. Sample of 50 random networks obtained by spatial random uniform distribution of the mobile devices and APs. For each plot, the red line gives the empirical mean $\hat{m}$ of the sample and the green dotted lines the interval $[\hat{m} - \sigma, \hat{m} + \sigma]$ where $\hat{m}$ is the empirical mean of sample and σ is the standard deviation.



(a) Stable matching resulting from Gaussian cost and BDAA.

(b) Matching resulting from the best-RSSI scheme.

Fig. 9. Comparison of the association obtained from (a) BDAA and (b) the best-RSSI scheme in scenario 2. These two figures show the AP at center (zoom).

means that BDAA gives a total throughput at the MAC level that is superior to the (unmodified) total throughput resulting from the maximization at the mechanism level (modified values) while the ratio was inferior to one in the modified case. This may come from the fact that in some cases, the equilibrium point (stable matching resulting from BDAA) may contain coalitions with lower modified worths (because of the penalization) w.r.t. those in the global maximum but higher worths at MAC level (real unpenalized setting).

To conclude, we compare our approach to the best-RSSI scheme in Scenario 2. The two matchings are compared Figure 9. We observe that the load is effectively shared among the APs and that the individual throughputs are greatly increased from 527 kbits/s when using best-RSSI to 1.64 Mbits/s for the coalition with AP1, 1.93 Mbits/s for the coalition with AP2, 2.59 Mbits/s for the coalition with AP3, 1.64 Mbits/s for the coalition with AP4 and 2.59 Mbits/s for the coalition with AP5. The individual performances are multiplied by a factor 3 to 5.

VII. Conclusion

In this paper, we have presented a novel AP association mechanism in multi-rate IEEE 802.11 WLANs. We have formulated the problem as a coalition matching game with complementarities and peer effects and we have provided a new practical control mechanism that provides nodes the incentive to form coalitions both solving the unemployment problem and reducing the impact of the anomaly in IEEE 802.11. Simulation results have shown that the proposed mechanism can provide significant gains in terms of increased throughput by minimizing the impact of the anomaly through the overlapping between APs. We have also proposed a polynomial complexity algorithm for computing a stable structure in many-to-one matching games with complementarities and peer effects. This work is a first step in the field of controlled coalition games for achieving core stable associations in distributed wireless networks. Further works includes for example the study of a dynamic number of users or the impact of interference.

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